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ABSTRACT

The reasons people most often give for the failures of U.S. schools involve poverty, racial inequality, and a host of social problems. This paper argues that even if all these conditions were remedied, the schools would not produce many more people with the ability to think than they do today. Teachers, who are usually able to think, do not know how they do it, and so they are not able to teach students how to think. An objective method has been created to get into the minds of thinkers and find out how they are doing the thinking. It is apparent that thinking is not knowledge, rather it is what thinkers do on knowledge to transform it. Teachers, because they are not aware of their own thought processes, generally teach knowledge and practical operations. Teaching mental operations is a special instructional task that requires special methods and techniques. These are analogous to algorithms, and may be thought of as human algorithms. To make all children good students and problem solvers, it is necessary to uncover unconscious algorithmic processes of expert learners and expert problem solvers and then describe those processes and the general processes of forming them explicitly. "Landamatics" is a name given to the algorithmico-heuristic theory and method of instruction based on these principles. It is a theory and method for creating expert performers in a systematic, reliable, and relatively fast way. An example of teaching mental operations through the Landamatics method is presented, and selected principles of the theory are summarized. (Contains two tables, four figures, and six references.) (SLD)

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ED 419 831

L. LANDA, Ph.D.

WHY SCHOOLS FAIL TO TEACH THINKING AND THE ABILITY TO EFFECTIVELY LEARN

AND

WHAT TO DO ABOUT IT

(The Landamatics solution)

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PART 1

The cry of business and industry everyone hears, but not many know what to do about it

Many newspapers recently wrote about a large New York bank that needed to hire 50 entry level employees. The bank interviewed several hundred school graduates yet could not fill all the vacancies.

Despite rather high unemployment, many companies still cannot find enough people who meet some basic requirements in terms of intelligence and ability to learn. The common complaint: high school graduates – even those who have good basic knowledge in different disciplines – don't know how to think, how to solve problems and make decisions.

Unlike industry in the past century, where physical skills were often enough to be a good worker, – the main ability that modern industry requires is the *ability to think* and *to learn*. Everyone, including teachers, realizes this. And still, schools are unable to produce students who are able to think and learn effectively. (There are of course exceptions.)

Why?

Is it enough to eliminate poverty and flood schools with money and modern technology?

The reasons most often given to explain the failure of schools to produce educated and intelligent people is poverty, racial inequality, insufficient funding, low teachers' salary, and lack of modern technological means of learning and instruction.

All these reasons do matter. All of them do have a negative effect on students, teachers and the learning process.

The question is this: Would our schools produce knowledgeable and intelligent people with highly developed thinking abilities were all those reasons eliminated?

Many would say, yes. I am saying, no. To improve material and social conditions is necessary but *not sufficient* for improving our ability to create people with intelligence.

I maintain that even if tomorrow our schools were flooded with money and all negative social factors were eliminated, our schools would not produce much more people with the ability to think than they produce today.

Why the elimination of negative factors and unlimited money won't by themselves produce more intelligent people

I was raised in an untroubled family. Not rich, but not poor. My parents were hard-working but not well-educated people. Because their poor education made them work hard for little reward, giving their children good education became one of the main goals in their life. They did their best to educate us and to motivate us to study as diligently as we could.

I was a highly motivated and conscientious student. I received good grades and teachers praised me for my assiduity and perseverance. Nevertheless, I was unhappy. Why?

Because I felt that I was lacking brains.

For example, I well knew all the rules of grammar, but made errors in writing. I could easily reproduce a teacher's reasoning in proving a geometry theorem but could never find a proof on my own. I knew all the laws of physics that were being taught but had significant difficulties in solving physics problems.

In other words, when I was a pupil, none of those negative conditions that are indicated today to account for the failure of American schools to effectively teach in general – and teach thinking, in particular – existed for me. And yet, my ability to think was impaired. I was a poor, a very poor thinker and problem solver.

I am sure that my teachers fifty years ago were as aware of the importance of developing students' thinking abilities as are teachers today, and did their best to teach us how to think. However, they definitely failed, at least in my case. And I was not alone. The majority of my classmates were in the same position as I. Knowing grammar rules, they also made errors in writing; knowing all appropriate geometric and other propositions, they also didn't know how to prove theorems and solve problems in other disciplines. Some of them – I know this for sure – were suffering from their inadequacy and perceived dumbness as much as I. And this dumbness wasn't just perceived. We *were* not very intelligent.

Were I and my schoolmates lacking brains? Why despite motivation and diligence didn't we know how to think?

Observation of a 13 year old boy which solves the mystery

When listening during a geometry lesson to the teacher's proof of a theorem, I, as always, didn't understand why, out of several known and applicable theorems, the teacher used one theorem in one case and some other theorem in another, similar case. I could *memorize* which theorem or theorems to use in proving a particular theorem, but I didn't know how to determine on my own which theorem(s) to use when solving a new problem. That is why I could *repeat* the process of proof that the teacher demonstrated to us, but I could not find a proof by myself.

After the lesson, I asked the teacher how I can know which theorem – out of many known – I should apply in one case or another. She said that this is impossible to explain and that this ability will come with practice. That later, when I will have acquired enough experience in proving theorems, my brain will come to know which theorem to use.

This explanation was not of great help to me and no consolation. It meant that I had to wait – who knows for how long – until my brain would acquire this ability.

I decided not to wait but to observe more closely how the teacher did the reasoning; maybe I could figure it out myself. At the next lesson, I concentrated on the language the teacher used in the process of proving a theorem. And something struck me immediately.

The teacher started the proof of a new theorem by saying: "We know that...". Who knows? *She* knows. I may not know this. Or, I in general may know this but I may not know why – out of all the things I know about geometric objects – I, in this case, have to use *this* particular knowledge and not some other.

I few minutes later, the teacher said: "It is obvious that...". Obvious to whom? To *her*, not to me.

In another several minutes, she said: "We see that...". Who saw? *She* saw. I didn't see it.

In short, I came to the conclusion that the teacher communicated to us the *results* of her thinking, not how she *came* to those results. In my today's language, I would say that she communicated to us the *results* of her thinking, not the *process* of thinking, although her reasoning looked as a process. But this was an *external* process ("it is obvious...", "we see...", "let us apply this theorem", etc.), not the *internal* process of thinking. The process presented to us was a *sequence of theorems* somehow *selected* by her to make a proof, but not the *sequence of thoughts* that led her to *selecting* one or another theorem.

Did other teachers teach the process of thinking?

Struck by this discovery, I began to observe how other teachers taught us. It turned out that they did the same things.

The teacher of Russian, when I made a mistake, usually asked me: "Don't you know this rule?" (She indicated the rule.) Of course I knew it.

What I didn't know was this: why, out of the dozens of grammar rules that I knew, I had to apply in *this* case precisely *this* rule and not some other rule? The cause of my error was not that I didn't apply any rule but that I applied an inappropriate rule. Not a *wrong* rule (all rules by themselves are correct), but precisely *inappropriate*. But how to know when to apply which rule? Which rule is appropriate for which situation?

By indicating the rule to be applied in this *particular instance* (this particular word, this particular sentence) the teacher taught me the connection between the *specific* rule and some *specific* and *singular* situation. And I could memorize this specific and singular connection.

But how to know which rule to apply in some *other* situation? In *any* possible situation? The teacher somehow knew how because she made the selection of proper rules correctly.

Obviously, she might have had some *general method* of rule selection by which she made the selection correctly in *any* situation. But she didn't teach us that general method. She didn't teach us how to think, to reason, in order to make a selection in any situation. She simply gave us the *results* of her *selection process* – the rule she had *already* somehow *selected*.

In physics and all other disciplines, I found the situation to be the same. When I couldn't solve a physics problem, our teacher used to say: "In fact, it is very simple. There is a rule (a law)... Do you know it? (Of course, I knew.) What does the rule (law) say? What follows from it? (I answered; the conclusion was obvious.) Isn't it truly simple?"

It was really very simple *after* the teacher *indicated* the rule (law) to be applied. But my problem was not that I didn't know how to draw a conclusion from the rule (law); the rule (law) explicitly contained that conclusion.

My problem was different: how to know which rule (law) – out of the dozens that I knew – I had to select and apply in one case or another. But exactly this my physics teacher didn't teach me: *she* made the selection *for me*, presenting me with the *results* of his selection process. But, from knowing the results of her selection process (the rules and laws she indicated to me), I didn't learn how to make the selection myself.

I remained a poor problem solver. I continued to feel that I was inadequate. I began to think about dropping out. I didn't do this only because I knew it would make my parents extremely unhappy.

Why teachers don't teach students how to think believing that they do so

While still in high school, I decided to become a psychologist in order to professionally study whether it was possible to make "dumb" students smart by changing the ways they are taught.

After graduation from the university, I went on to a graduate school with the objectives of my Ph.D. dissertation formulated well in advance:

(1) to discover how teachers themselves think; what are the *internal* thought processes they go through when selecting and applying knowledge and solving problems;

(2) to understand why they don't teach students their own internal thought processes and methods of thinking;

(3) to figure out the methods of instruction that would teach students not so much the *results* of thought processes but the *processes* themselves.

But in order to figure out the methods of instruction, it was necessary to come to know how teachers themselves thought and solve problems.

How the teachers think: dialogs with teachers

How to come to know the processes of thinking? It seemed simple: go and ask the teachers about how they think.

I put together a list of questions and began interviewing teachers. I started with teachers of mathematics.

Questions like "Would you describe what you do in your mind when you are looking for a solution to a problem?" turned out to be futile. Nobody could answer them, or the answers were very general and unspecific. For example: "I analyze the problem". "I analyze the conditions". "I compare the problem with others whose solution I know". "I gather all the information I can from what is contained in the statement of the problem", etc.

Of course, they did do what they said, but they did not say *how* they did so. Insufficiency of such a description is clear if we imagine a person who does not know how to solve a problem and he is given instructions like "Analyze the problem thoroughly", "Compare this problem with others whose solution you know", "Gather all the information contained in the conditions"; and so on.

Such instructions would be of marginal help. The actual problem is this: "How to analyze?", "With which other problems do I have to compare this one?" (I have solved many dozens of them in the past), "How to make the comparison?", "What information to gather and how?" (I seem to have gathered all I could), etc.

Since teachers were unable to answer my rather general and open-ended questions, I decided to give them a number of geometry problems, have them solve them and, in the process of solution, tell me what they were doing in their minds.

To my great surprise, I heard essentially what I was hearing when I was a student in the high school: "It is obvious that...", "We see that...", and the like.

But now I could ask them questions which I could not ask in my childhood as a student.

Dialog 1.

I; You said it is obvious. What makes it obvious? To me, for example, it is not obvious. What do you do in your mind which makes it obvious to you"?

Teacher: This I can't explain. I read the problem and certain things are immediately obvious to me.

I: But when *I* read the problem, these things are not at all obvious to me. What do you do in your mind with the conditions of the problem – which I don't do – that makes certain things obvious?"

T: I don't do anything in my mind. I just read the problem.

I: But I also read.

T: If you had such the experience I have, then it would become obvious to you too.

I: So I should wait until experience will develop something in my head so that certain things would become obvious to me as they are obvious to you?

T: I don't know any other way.

Dialog 2

I: You said, "This chord is as a side of this inscribed triangle. Let us use this theorem...". How did it come to you mind to use this cord as a side of a triangle"?

Teacher: I just saw it.

I: How did you see it? What did you do in your mind in order to notice it? This chord is an element of some other figures as well.

T: A strange question, how I saw it. I *looked* at it and I immediately *saw* it.

I: But I also looked at it, however I didn't see it. This means that you did something in your mind which I didn't do. What did you do in your mind that allowed you to see what you saw?

T: Nothing.

I: Why then you could see and I couldn't?

T: Because you don't have my experience. If you had my experience, you would also see what I saw.

I: But experience itself cannot solve problems. Experience creates some *processes* in your mind which enable you to see what I can't see. What are those processes? What do you *do* in your mind when looking at a diagram which makes it possible for you to see *more* in the same things than I see?

T: I don't know of any processes. Of course something should be going on in my mind but I never thought of it.

I: Do you believe it is possible to uncover and teach these processes?

T: I don't know. I doubt. Those processes come with experience. How can you teach experience?

I: I am not talking about teaching experience. I am talking about teaching processes which today are formed in the course of experience.

T: You cannot teach what only experience can create.

Dialog 3

I: You have easily solved the problem but told me very little of what you did in your mind which had led you to the solution.

T: How can I tell it? Everything flies in your head so fast.

I: Can you think retrospectively of what you were doing in your mind when solving this problem?

T: It is very hard.

I: Can one teach the processes that you perform in your head and thereby create problem solvers as good as you are?

T: Certain things you can teach. But to become a good problem solver, you have to have brains.

I: What's "brains"? Isn't it some processes that you execute in your head? The static brain itself does not solve problems. It is the *processes* that you perform in your brain that make up the solution process. Can one teach those processes?

T: As I said, you can teach certain things, but you cannot create brains. This is from God. And of course you have to have experience. You can't teach experience.

I: Can you teach the processes that are formed in the course of experience and thus reduce the time of experiencing?

T: You can't create brains and you can't substitute teaching for experience. If a person is dumb, he will remain dumb no matter how you teach him.

The discovery of a naive researcher: teachers don't know how they think

I was naive enough to believe that in order to come to know how teachers think it was enough to ask them about it. The striking discovery I made was that they *were able* to think but they *didn't know how they did it.*

This led to the explanation of why teachers can't effectively teach students how to think. If one is unaware of certain processes – *any* mental processes, not just thought processes – he/she cannot effectively teach them. And if you don't teach processes purposefully and effectively, their formation *in some students* proceeds spontaneously in the course of experience.

The teachers were quite right pointing out to experience as a critical factor in forming the ability to think. But experience is a random, spontaneous process that sometimes leads to developing in people the ability to think but sometimes – more often – doesn't. Everyone knows people who have lived a long life and have had a lot of various experiences and still remain not very intelligent and smart.

What is experience?

If you don't know how to do something (to turn on an unfamiliar device, to unlock a door when keys are lost – or to solve any other problem), you use the *trial and error method.* This is what students and others do when they don't how to solve a problem. But some people are able to learn – and do learn – from their errors, many others aren't able – and don't.

We can offer the following definition of experience:

Experience is nothing other than years of trial and error whereby some people discover the mental processes which they were not taught in the course of instruction and which enable them to effectively perform tasks, solve problems and make decisions.

Can thought processes be taught and developed systematically and reliably – without the need for years of experience?

This was the question I was to answer in my study. But to answer it, it was necessary to know the thought processes of teachers – we will now use a more generic term *expert thinkers* – and then try to *replicate* those processes of expert thinkers in the minds of students.

If the replication was successful, then the answer to the question would be positive. If not, the question would remain unanswered: the reason for failure might be not the fact that expert thinkers' thought processes are not replicable but that we didn't know how to effectively do it.

How to get to know thought processes of expert thinkers when they can't describe them?

This was now the main question. Because thought processes are unobservable, and expert thinkers are largely unaware of them, it was necessary to devise an *objective* method of gaining insight into the mental processes of expert thinkers. One that would allow us to get inside the minds of expert thinkers and *uncover* the processes of which they themselves were unable to give an account.

Was it possible?

Analysis of methods used in other sciences showed that, in principle, it was possible. Indeed, the atomico-molecular processes within material objects were unobservable and those objects could not tell scientists what was going on inside them. Nevertheless, physicists managed to develop objective methods of penetrating matter and, as a result, could discover those processes.

If this was possible in physics, then why not in psychology?

And we created an objective method – a set of techniques – of getting inside the minds of expert thinkers and uncovering the nonconscious mental processes of which they themselves were unaware.

It is impossible in a short article to describe this method. Moreover, for the purpose of this article, the knowledge of how the method works is not important. More important in this context is *what* was discovered using the method.

But before we discuss what was discovered, it is necessary to become clear about some prerequisite notions.

Knowledge and thinking

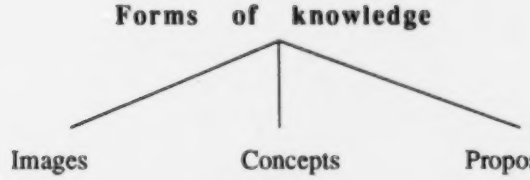
Knowledge

Everyone knows what *knowledge* is: it is a reflection in our minds of objects or phenomena in the form of images, concepts or propositions. To know something means to have an image of that object, and/or a notion of it, and/or be able to state a proposition about it.

When one closes one's eyes, one can see in the mind a *picture* of certain objects, for example, of a pencil. This picture is an *image*.

When one is asked to *explain* what is a pencil and a person is able to verbally *indicate (list)* the characteristic *features* of a pencil, it means the person has a notion (concept) of it. (Whether correct or incorrect is a different matter.)

When someone is asked to *give a definition* of a pencil or *make a statement* about its attributes, and he is able to do so, then it can be said he has knowledge of a pencil in the form of a proposition.



It is well known that people may have knowledge of certain objects in the form of images but not in the form of concepts (one may *depict* the objects, make a *drawing* of them but have difficulties in isolating and verbally listing their characteristics).

It is also possible that a person may have knowledge in the form of concepts (i.e., be able to list objects' characteristics) but not be able to give their logically correct definitions or formulate their related rules .

A person of course may have the knowledge of objects in all three forms.

Thinking

Everyone knows that a person may be very knowledgeable but not very smart. And vice versa: he may be smart but not very knowledgeable. This makes it obvious that knowledge and thinking are different things.

What is the difference?

A simple answer is this: thinking is not knowledge, it is *operating* on (or with) knowledge which allows one to derive from the given knowledge some other knowledge (for example, to create a new image, to draw a conclusion, etc.).

What does it mean to operate on knowledge? More generally, what operating means?

Physical actions

The notion of operating implies some *actions*. To operate a plant, for example, means to *do* certain things on the plant's objects in order to achieve certain production or other goals.

Operating consists of certain *actions*: for example, to *turn* on a machine, to *set* a certain regimen of its work, etc.

The actions (or "operations*) we just mentioned are *physical* (*practical*) actions.

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*We will be using the terms "action" and "operation" interchangeably.

The notion of physical actions is simple and everyone has it: physical (practical) actions deal with *material, tangible* objects and they bring about their physical, material changes, or *transformations*. Sharpening a pencil, putting it on a table, turning it around are different *transformations* of a pencil caused by different *actions* performed on it.

However, the notion of an action is not limited to physical (practical) actions. For example, talking with a person about his/her thinking processes, we may ask the question: "What did you *do* in your mind that led you to that decision?"

What does "to *do* something *in your mind*" mean?

Mental actions

Doing something in the mind definitely implies some *actions*. But the mind does not have hands, and it can't perform physical actions that transform material, tangible objects because they are not sitting in the mind. What kind of actions is then meant when we speak of doing something in the mind?

These actions are *mental*.

But designation of such actions as mental does not explain much. It is still not clear what an action in reference to the mind means.

Let us go back to our pencil. By sharpening it, we change (*transform*) its mass and shape, by physically turning it around we *transform* its physical spatial orientation.

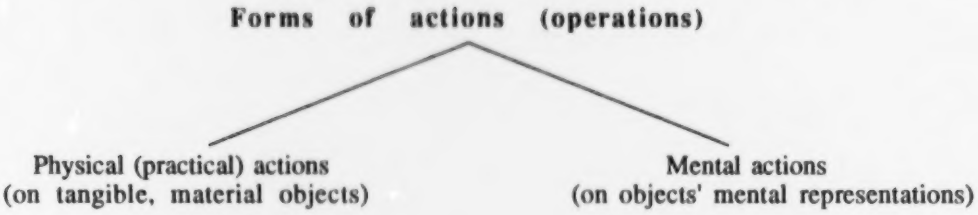
Now close your eyes and picture the pencil in your mind. You have an *image* of the pencil which is knowledge in the form of an image. Can you now *turn around* in your *mind* the mental picture (*image*) of the pencil in the way similar to how you did it physically with the material tangible pencil?

Everyone can.

What happened to the image? It *changed*. The original image of a pencil (in the initial position) was *transformed* into another image (the image of the pencil in another position).

Thus, we can transform not only the tangible, material things – by means of *practical, physical* actions. We can also transform the images – by means of *mental* actions.

Mental actions are thus *real* actions, since they bring about *real* transformations. But these are transformations (in our example) of *images* of material objects rather than of objects themselves.



Examples of mental actions (operations) on images

By means of mental actions, we can do with images (or, on images) dozens if not hundreds of things. For example, having an image of an automobile, we can:

break the image *apart*;
isolate certain parts in the image;
switch parts (for example, mentally put one wheel in place of another);
move the car in the image in any direction;
abstract car's attributes in the image (for example, color);
replace attributes in an image (e.g., replace red color by blue color, and thus turn the red automobile in our imagination into a blue one);
compare an image of a car (or any of its parts) with some other image.
Etc. Etc.

However, mental actions can be performed not only on images but on concepts and propositions as well.

Examples of mental actions (operations) on concepts and propositions

On concepts we can:

isolate characteristic features (make them stand out in our mind)
remove a feature(s) from the set of features;
add a feature(s) to the set of features;
replace one feature by another one;
compare features;
anticipate presence of certain features on the basis of existing features;
assess the necessity of a feature for a certain conclusion;
assess the sufficiency of a feature for a certain conclusion;
assess the probability of a feature provided some other features are present;
find features of a higher (or lower) order of generality;
structure features in a certain way;
convert features into propositions (e.g., formulate a definition on the basis of some given features).
Etc. Etc.

On propositions we can:

change them;
replace them by other propositions;
compare them;
draw conclusions from given propositions;
assess the truth (or falsehood) of a proposition.
Etc. Etc.

Why knowledgeable people may be not smart, and vice versa

Thus, thinking is not knowledge; it is what we *do* on knowledge, what *actions* (operations) we perform on it and how we *transform* it.

One can have in his mind a large repertory of images, concepts and propositions but a small repertory of mental actions. As a result, he will not be able to perform on knowledge all the actions that need to be performed in order to solve varied problems and make varied decisions. Such a person will be knowledgeable but not smart.

Another person may have in his mind a large repertory of mental actions but a small repertory of images, concepts and propositions. If his mental actions are well generalized, he can successfully apply them not only to the knowledge he already has but to any *new* knowledge he receives or deals with. Such a person will be not very knowledgeable, but smart.

This, by the way, explains why, for example, a good minister of transportation may become a good minister of agriculture without having a wide and specialized knowledge in either of those areas. The repertory of mental operations is so large and the level of their generalization is so high in such people that they can effectively transform *any* knowledge with which they are confronted.

There may be other situations.

For example, a person may have in his mind a large repertory of both knowledge (images, concepts, propositions) *and* mental actions. If, however, he does not know which mental action to apply to which knowledge in order to solve problems and make decisions, he will also be knowledgeable but not smart.

Or, a person may have in his mind a large repertory of both knowledge and mental operations, and also know which mental operation to apply to which knowledge, but not be able to execute the mental operations.* He will also be knowledgeable but not very smart.

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*Analogy with the physical actions: a person may know how to lift a weight but not be able to do this. Or, he may know what to do with his skis and body to make a turn while descending from a hill, but not be able to perform those actions.

Still another person would have in his mind a large repertory of both knowledge and mental operations, know which operation to apply to which knowledge, and also be able to execute his mental operations. However, if his operations are not well generalized, he would apply his mental operations not to *all* the knowledge to which they should be applied but only to some *subset* of that knowledge. Such person will also be knowledgeable, but his intelligence will be limited to those objects (areas) where his mental operations will be applied. This is determined by the nature and degree of generalization of his mental operations.

What experts are aware and unaware of

Our studies have shown that expert thinkers and performers (in any area of activity) are much more aware of the *knowledge* they use in solving problems and making decisions, and the *physical (practical) operations* involved, than of the *mental operations* they execute in their minds. Specifically, experts are aware of approximately 30% of the mental actions they perform in their minds while solving problems and making decision. Their greater awareness of knowledge and physical operations does not mean that they are aware of *all* knowledges and physical operations they use in performing their tasks. The only point we are making here is that the awareness of mental operations is much lower than the awareness of knowledge and physical operations.

Why schools want to teach thinking but do not do so

It is precisely because teachers – as all other experts – are largely unaware of their mental operations and processes and, as a rule, are not instructed, in the course of teacher training, in the mental operations involved in learning and problem solving.*

This explains why teachers primarily teach knowledge and physical (practical) operations, but not the mental operations. If, however, students are taught only a portion of the mental operations (about 30%) of which teachers themselves are aware of, then it is clear why students have difficulties in learning and problem solving. They simply don't know what to mentally *do* on available knowledge in order to solve problems and make decisions. They may want to think but they don't know *how*.

Why some teachers teach better than others

The awareness of approximately 30% of the mental operations is an *average* awareness. As with all average numbers, some teachers are aware of more than 30%, some others of less than 30%. The first naturally teach students more mental operations than the second. This makes learning and

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* This unawareness of many mental operations is thus not teachers' fault, as it is not a fault of experts in all other areas of activity who are largely unaware of their mental processes.

problem solving easier for his/her students and enables them to reach higher achievements. Of course, teaching a larger repertory of mental operation (and teaching them effectively) is not the only factor that makes a good teacher good. There is a number of other factors.

The relationships between all the factors can be expressed in the following way: The knowledge of mental operations and effective teaching them to students is not enough to make one a good teacher, yet without this knowledge and effective teaching of mental operations no teacher can be good. Although the factor under discussion is not a sufficient factor, it is a necessary one.

Teaching mental operations as a special instructional task requiring special methods and techniques

For a teacher to be able to teach thinking in a purposeful, systematic and reliable way, it is not enough to know *what* mental operations are to be taught to students in certain contexts. It is necessary also to know *how* to teach them.

The following well known fact may serve as an illustration: people are often able to perform certain physical (practical) actions but not their corresponding mental action. In whatever way the physical operations are formed (through an organized instruction or independent learning), the formation of physical operations does not automatically lead to the formation of their corresponding mental operations.

A characteristic example:

Geographical maps are oriented in such a way that north is always on the top of the map and south is at the bottom. When people drive from the south to the north using a map, they have no problems in taking guidance from it: if the map shows a right turn, you should turn to the right. However, when one drives from the north to the south, then, to take guidance from the map, one has to turn the map around in the head (i.e. to *turn around* the *image* of the map). Some people are able to execute this mental operation on the image, some others aren't. What the latter normally do is this: they *physically turn around* the actual *tangible* map. They know the mental operation they have to perform on their image of the map but are unable to execute it. The mental operation is too *weak* to do the job of turning around this image.

This example demonstrates one of the important qualities of mental operations – the degree of their strength. It is like the strength of muscles: one can lift 150 pounds, another one can't.

In order to teach thinking and develop students' thinking abilities, the teacher must know not only *what* mental operations are to be taught to students but also *how* to teach them. How, for example, to endow mental operations with the necessary strength and, if needed, enhance that strength.

In the framework of this introductory article, it is impossible to describe the methodology of teaching and developing mental operations. The

purpose of this and some other examples given here is to show how multi-faceted is the problem of teaching mental operations. Teacher students are often not even exposed in their training to many facets, and are often unaware of them, let alone not equipped with proper methods and techniques of handling different aspects of teaching mental operations.

To think, the knowledge of single mental operations is not enough

As a rule, thinking requires execution of *systems* of mental operations rather than disparate, single operations. Therefore, in order to effectively teach thinking, it is necessary for the teacher to know those systems.

Each type of problem requires its own specific system of operations. The mental operations involved in applying rules are different from the mental operations involved in building hypotheses. The mental operations involved in solving an arithmetic problem are different from those required to make a scientific discovery or a technical invention.

Despite the large diversity of mental operations and their systems, they can be grouped and combined in classes (categorized). As everything in the world, a system of mental operations – *thought processes* – can be categorized on the basis of different principles. We will consider here the classification of thought processes on the basis of *objective requirements* which problems to be solved make to the mental processes of their solution.

Algorithmic problems, algorithmic mental processes, and algorithms

When you have to start a car or an aircraft, you have to execute a certain number of operations in a certain sequence.

If you fail to execute at least one of the operations, or you perform them in a wrong sequence, you won't be able to accomplish your task.

When you work with the computer or any software – or with a myriad of technical devices or industrial processes – you are in the same situation.

If you fail to execute at least one of the operations or you perform them in a wrong sequence, you won't be able to accomplish your task.

The examples just given refer to practical actions involved in performing tasks. However, we find the same situation with a great variety of tasks or problems which require mental actions for their solution.

Here is an example from mathematics that everyone knows:

To divide one (larger) whole number (dividend) by another (smaller) whole number (divisor), one has to:

1. Check to see how many digits the divisor has.

2. Separate the same number of digits in the dividend.

3. Check to see if the separated number in the dividend is greater than the divisor.

If not, separate the next digit in the dividend and proceed to instruction 4.

If yes,

4. Do the division.

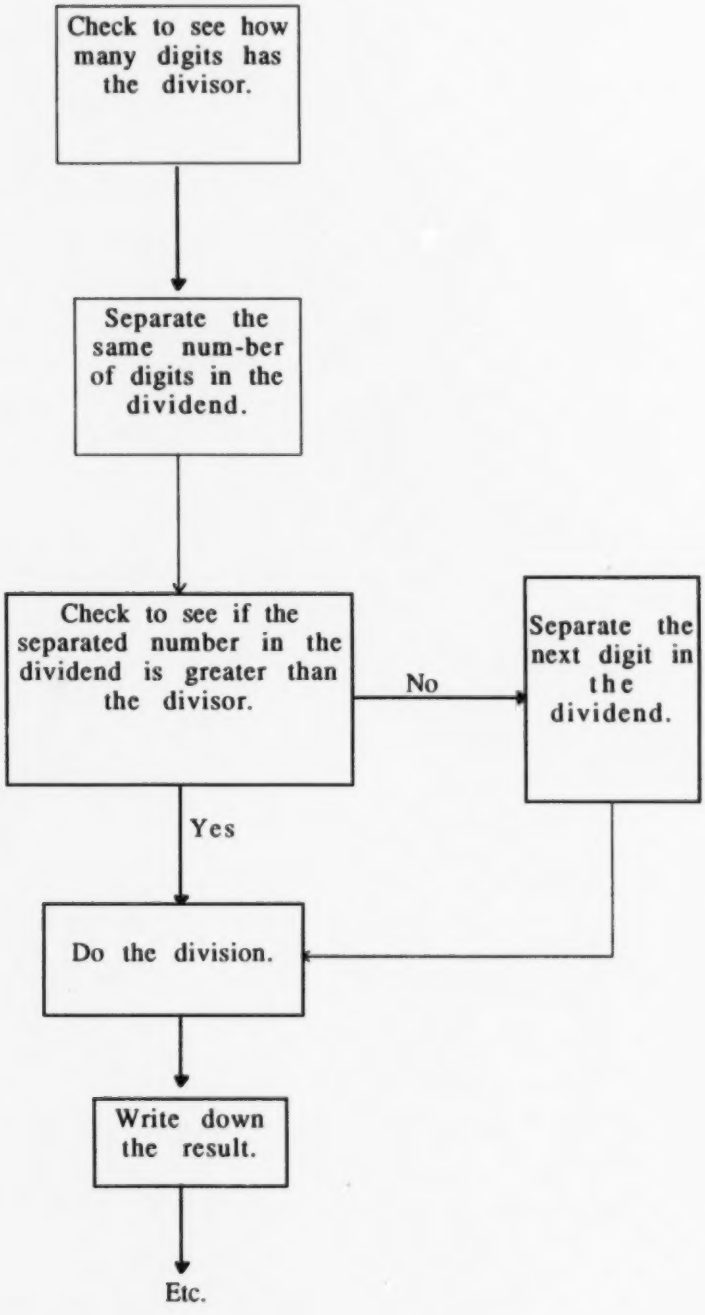
5. Write down the result.

Etc.

This process can be more conveniently presented in a flowchart form. Here is a fragment of such a flowchart representation.

How to divide one (larger) whole number by another whole number (a fragment)

In order to do the division:


Check to see how many digits has the divisor.
Separate the same num-ber of digits in the dividend.
Check to see if the separated number in the dividend is greater than the divisor.
No
Separate the next digit in the dividend.
Yes
Do the division.
Write down the result.
Etc.

This prescription is commonly known as an *algorithm of long division* which in many countries is normally taught in the primary school.

The described algorithm is a precise prescription (a set of instructions, commands) as to what one should *do* with two whole numbers in order to divide one (larger) number by another (smaller) number.

This algorithm is:

(a) unambiguous and easy to understand and follow;

(b) applicable to *any* two whole numbers (in this sense, it is a *general* procedure, a *general* method of solving problems of this type), and

(c) guarantees the results (arrival at the solution) if one performs all the operations indicated in the algorithm in the proper sequence.

If one fails to perform all the operations indicated in the algorithm or performs them in a wrong sequence, he/she will not be able to arrive at the sought-for result.

From the notion of an algorithm as a *prescription* (a set of *instructions*) as to what to do in order to solve any problem belonging to a given class, one has to distinguish the notion of an algorithmic *process* as a set of *operations* themselves. An algorithm, as a set of *instructions*, determines the set of *operations* – an algorithmic *process* – to be performed in order to solve a problem.

Problems that *require* an algorithmic process for their solution are *algorithmic problems*.

There may exist more than one algorithm for solving one and the same problem, but an algorithmic problem cannot be resolved without performing one of the required systems of operations (physical and/or mental), i.e., without executing an appropriate algorithmic process.

There are two ways in which people can solve algorithmic problems: by executing a known algorithmic process or by trial and error.

In the latter case, a person in the process of trying different operations *stumbles upon* the required operations in the required sequence. This means that he has *discovered* an appropriate algorithmic process. If he *remembers* the discovered operations (and their sequence) and can *verbalize* them, then it can be said that he has discovered not only the algorithmic process but the algorithm (i.e. the algorithmic instructions) as well.

(To be continued in part 2)

PART 2

The revolutionary role of algorithms in teaching, learning and doing mathematics

The algorithm of long division referred to previously was discovered (found) by a Persian mathematician Al-Khowarizmi in the 9th century. Prior to the discovery of this algorithm, to divide one large number by some other large number was a difficult task which only intelligent and knowledgeable people could perform – and it often took them a lot of time.

After the discovery of the algorithm, it became possible to teach even small children how to divide numbers of any size. Young children equipped with the algorithm can do division much faster than any adult – even an intelligent one – who does not know the algorithm.

The discovery of algorithms became one of the main objectives of mathematics as a science. On the other hand, the discovered algorithms have completely revolutionized both the teaching and learning of mathematics, as well as its doing.

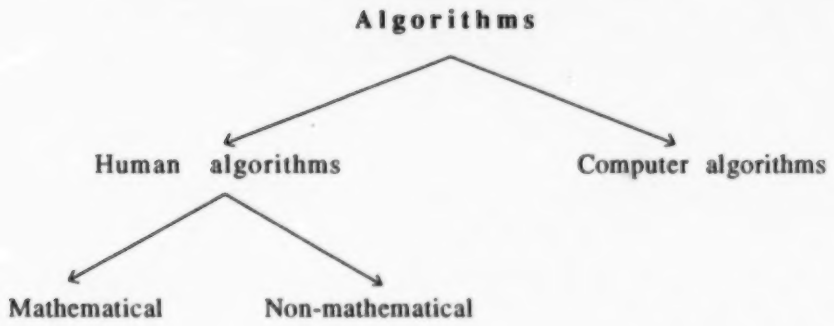
Mathematical and non-mathematical algorithms

Until the mid '50s, an algorithm was a mathematical notion, for it was believed that algorithms exist only in mathematics. .

Analyzing, at the beginning of 50's, how people solve problems in different disciplines, the author of this paper discovered that processes similar to the algorithmic processes in mathematics exist for solving not only mathematical problems. Such processes are used by people for solving problems in *all kinds of disciplines and areas of activity* including physics, chemistry, economics, medicine, social sciences, linguistics , music, and sports.

I *generalized* the notion of an algorithm and termed algorithms used by people for solving all kinds of problems *human algorithms*. With this generalization, the mathematical algorithms became just a particular case of human algorithms. Later, with the advent of computers, the notion of algorithms began to be used in computer science. Algorithms for programming computers were called in computer science *computer algorithms*.

A classification of algorithms, which comprises all kinds of algorithms, can be presented in the following way:



Although all algorithms have certain features in common, human algorithms differ radically from computer algorithms. This explains why computers are unable to work on the basis of human algorithms. And, vice versa, why humans cannot solve problems following computer algorithms. Theoretically, humans are able to follow computer algorithms but, practically, doing so is alien to the way human way people think and can drive a person crazy.*

In this article, we will be dealing only with human algorithms.

Why till now algorithms are used predominantly in teaching mathematics

The reason most people (including teachers) are unaware of the algorithmic processes they go through in solving all kinds of non-mathematical problems is that algorithmic processes are largely *unconscious*. But if a person is unaware of his algorithmic processes, she is unable to verbalize them. This in turn means that she doesn't know the algorithms that correspond to her algorithmic processes. It is obvious that if a person (teacher or not) does not know something – including algorithms – she cannot teach it.

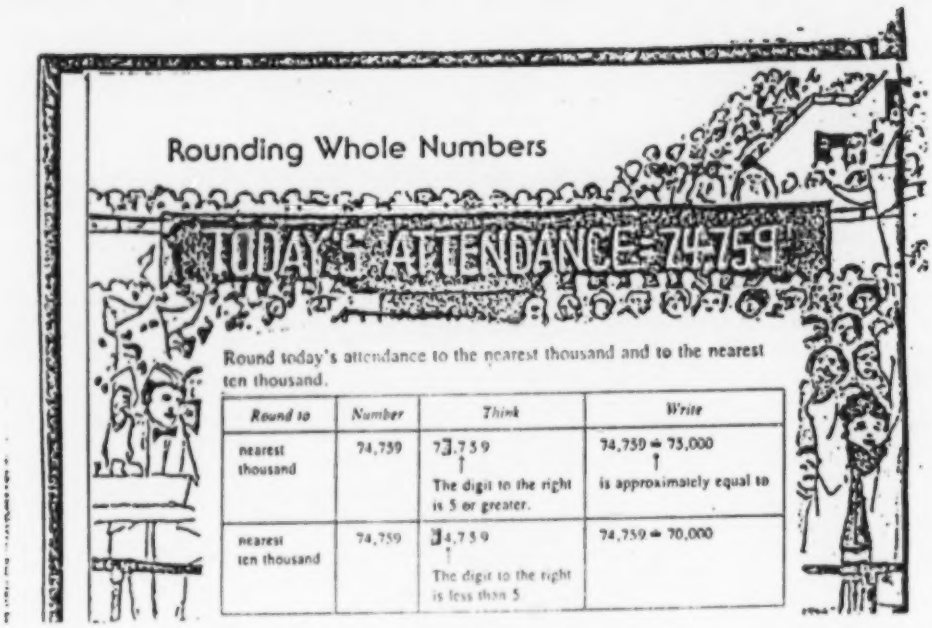
Even in teaching mathematics the instructional potential of algorithms is often not used

Students, for example, experience difficulties in learning how to round whole numbers.

Here is a an explanation of rounding from "Holt Mathematics":

---

* In more detail, the difference between mathematical, computer and human algorithms is examined in Landa's "Algorithmization in Learning and Instruction" and in "Instructional Regulation and Control: Cybernetics, Algorithmization and Heuristics in Education", both published by Educational Technology Publications, Englewood Cliffs, New Jersey.



Round today's attendance to the nearest thousand and to the nearest ten thousand.

Round to	*Number*	*Think*	*Write*
nearest thousand	74,759	7[4].759 ↑ The digit to the right is 5 or greater.	74,759 ≈ 75,000 ↑ is approximately equal to
nearest ten thousand	74,759	[7]4,759 ↑ The digit to the right is less than 5	74,759 ≈ 70,000

This explanation is a challenge even for adults. More important is that from this explanation is not clear what one has to do – and in what sequence – in order to round a number. A student has to figure it out by himself which not everyone is able to. That is why so many students don't know how to do rounding or have difficulties in doing it. Some never learn.

Here is a simple algorithm for rounding, designed by applying the algorithmic approach to teaching and learning:

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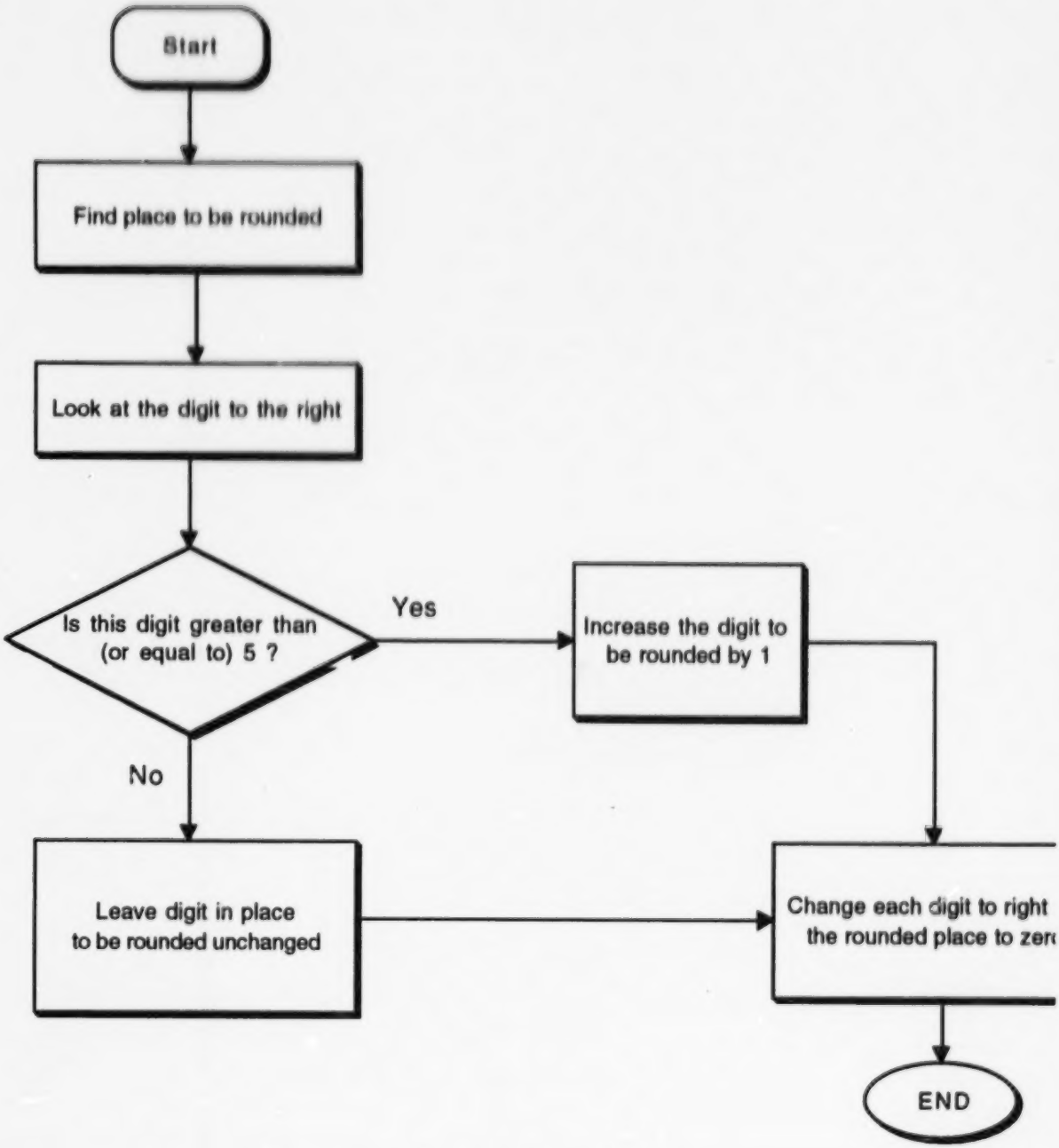
Algorithm

for rounding a whole number


Start
Find place to be rounded
Look at the digit to the right
Is this digit greater than (or equal to) 5 ?
Yes
Increase the digit to be rounded by 1
No
Leave digit in place to be rounded unchanged
Change each digit to right the rounded place to zer
END

To see the effectiveness of the algorithm-based instruction, which teaches algorithms, as compared with the conventional instruction which is represented by explanation in "Holt Mathematics", a simple experiment can be conducted. Break down a group of students, who know what rounding of a whole number means but who cannot do the rounding, into two groups. Give one group a copy of the page with the Holt Mathematics' explanation of rounding and a few tasks to perform. Give the other group the algorithm and the same tasks. Then compare how students in each of the group learned and performed rounding. The results will speak for themselves.

Some examples of non-mathematical algorithms

Example 1. How to determine the type of a lens.

Here is how the characteristics of lenses are explained in a physics textbook:

> **Kinds of Lenses.** There are two basic kinds of simple lenses. Light bends when it passes through either kind. Lenses that are thicker in the middle than at the edges are called *convex*, *converging*, or *positive*. Parallel light rays passing through a convex lens bend inward so that they meet at a point on the other side. Lenses that are thicker at the edges are call *concave*, *diverging*, or *negative*. Parallel rays passing through a concave lens bend outward so that they spread apart.
>
> **A Plano-Convex Lens** has one plane surface and one convex surface. It is used in certain types of slide projectors.
>
>
>
> **A Plano-Concave Lens** has one plane surface and one concave surface. It is combined with other lenses for cameras.
>
>
>
> **A Double-Convex Lens** has two convex surfaces. It is used in various magnifying glasses.
>
>
>
> **A Double-Concave Lens** has two concave surfaces. It is used in reducing glasses.
>
>
>
> **A Concavo-Convex Lens** has one convex surface of greater curvature than the concave surface. It is used to help correct farsightedness.
>
>
>
> **A Convexo-Concave Lens** has one concave surface of greater curvature than the convex surface. It is used to help correct nearsightedness.
>
>

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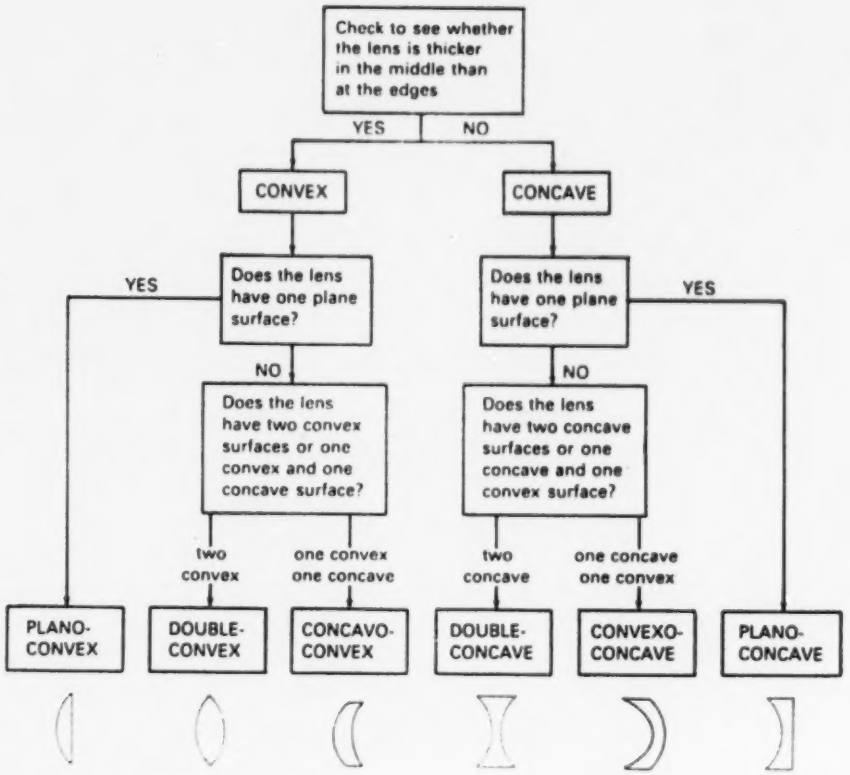
From this explanation, it is not clear what one should mentally *do* in order to identify the type of a lens. As a result, having understood the explanation, many students fail to do the identification or make errors. The cause of failure or errors is this: they don't know how to *convert* the *knowledge* of lens characteristics into a *system of mental actions* (an algorithm of identification) required to perform the process of identification.

Here is an algorithm for identifying the type of a lens developed by the author. It clearly and unambiguously indicates what one has to *do* in his/her mind in order to solve the lens identification problem – and to solve it easily and fast:*



* A detailed description of how to conduct an algorithm-based lesson that would teach not only knowledge of lens characteristics but the algorithm of lens identification as well is given in our article "A Fragment of a Lesson Based on the Algo-Heuristic Theory of Instruction" in the book "Instructional Theories in Action" edited by Charles M. Reigeluth. Lawrence Erlbaum Associates, Publishers, Hillsdale, New Jersey, 1987.

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To see the effectiveness of the algorithmic approach to teaching lenses vs. the conventional approach, one can conduct an experiment with two groups of students similar to the one described for rounding the whole numbers.

Example 2. How to make a decision which word, out of a group of synonyms, to choose in one case or another.

Learning of any foreign language involves learning how to make an ocean of decisions (a fact which is not realized by proponents of the direct method of teaching foreign languages).

For instance, how a learner of English as a foreign language would know – and decide – in which case he should say "I *did* this" and in which case, "I *have done* it". This difference in tenses does not exist in many languages.

Or, how to know – and make a decision – when one should use the verb "to do" and when, "to make". The difference between "to do" and "to make" does not exist in a number of languages.

Here is an example we will examine in more detail:

There are three English verbs: "to offer", to suggest" and "to propose".

The meanings of these three synonyms are conveyed:

In Russian – by a single word: *predlozhit'*.

In German – by two words: *anzubieten* and *vorzuschlagen*.

In English – by three words: *to offer, to suggest,* and *to propose*.

Examples

Russian	English
1. I want to *predlozhit'* you an interesting book.	1. I want to *offer* you an interesting book.
2. I want to *predlozhit'* you to take a bus.	2. I want to *suggest* that you take a bus.
3. I want to *predlozhit'* a more decisive course of action.	3. I want to *propose* a more decisive course of action.

Problem: How can, for example, a Russian student of English know in which case instead of *predlozhit"* he should use "offer", "suggest" or "propose"?

This was an actual problem for the author of this article when he began to study English. I asked many teachers of English, but no one could give an explanation that would allow me to make correct decisions. Very often their

explanations led to errors, if I were to follow them. The inability to explain was characteristic of the native speakers of English as well (the British and the Americans). Normally, they choose the right verbs automatically, without any difficulty or error, but none of them was able to explain to me how they made their unconscious choices (or decisions). Were I to follow native speakers' explanations, I would make errors. In my discussions with the teachers and native speakers, I tested each of their explanations by examples, and each teacher and native speaker immediately saw how, following their explanations, I came up with errors. Of no help were also a number of English textbooks and dictionaries.

So I had been making errors for 8 years, often using inappropriate verbs.

This lasted until I. Pavlova, one of my disciples and a graduate student at the Moscow Institute for Teaching Foreign Languages, decided to choose developing algorithms for making word-selection decisions as a topic of her Ph.D. dissertation. Among the algorithms she developed was an algorithm for choosing the right word among synonyms under discussion.

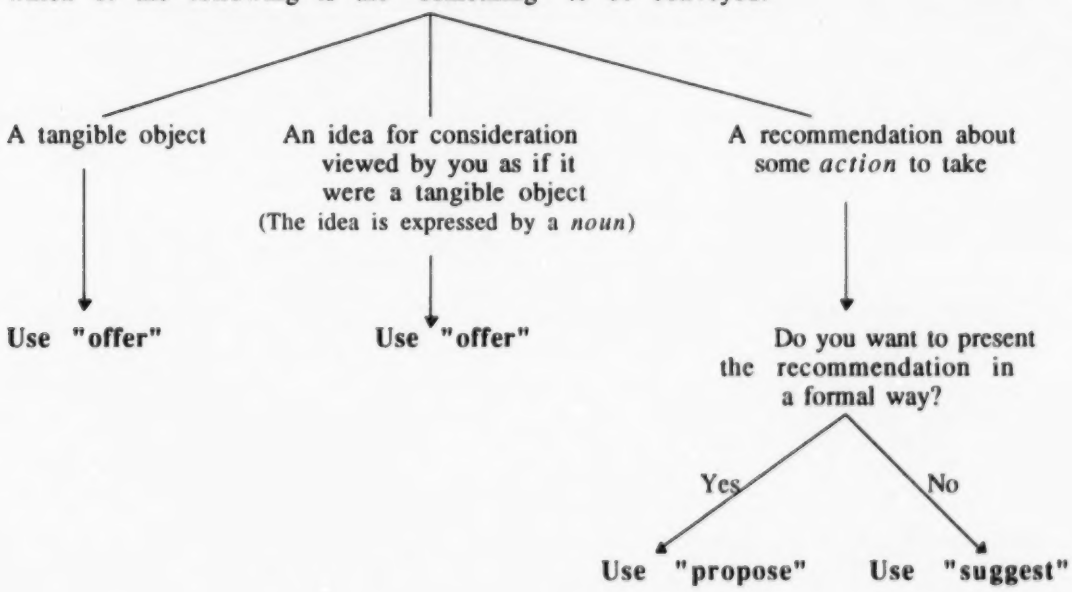
Here is her algorithm slightly modified by us:

Algorithm

for choosing among "to offer", "to suggest" and "to propose"

All three English verbs have one thing in common (that is why they are synonyms): they mean that someone conveys something to another person or a group of people.

In order to decide which verb to use in one case or another, check to see which of the following is the "something" to be conveyed:


A tangible object
An idea for consideration viewed by you as if it were a tangible object
(The idea is expressed by a *noun*)
A recommendation about some *action* to take
Use "offer"
Use "offer"
Do you want to present the recommendation in a formal way?
Yes
No
Use "propose"
Use "suggest"

Let us test the algorithm:

Problem situation 1: "I want to *predlozhit'* you ten dollars": to *offer*?, to *suggest*?, or to *propose*? Ten dollars is a tangible object.

Answer (following the first branch of the tree): I want to *offer* you ten dollars.

Problem situation 2: "I want to *predlozhit'* a suggestion": to *offer*? to *suggest*? or to *propose*? A suggestion is an idea for consideration that I view as a tangible object ("suggestion" is a *noun*).

Answer (following the second branch): I want to *offer* a suggestion.

Problem situation 3. "I want to *predlozhit'* that you go to Washington by plane": to *offer*? to *suggest*? or to *propose*? I am giving a recommendation about some *action* to take.

(We have to follow the third branch.)

Do I want to give this recommendation in a formal way (say, to a foreign delegation)? If yes, then "propose": "I *propose* that you go to Washington by plane".

If I give this recommendation not in a formal way (say, to a friend), then "suggest": "I *suggest* that you go to Washington by plane".

For years I was making errors in choosing the right verb. Now, with this algorithm, everyone can teach a student of English, within 5 to 10 minutes, how to make right choices. Additional minutes of practicing will lead to *internalization* and *automatization* of the operations of this algorithm in the learners' minds, as a result of which learners will not need to recall the algorithm. Operations of the algorithm will be carried out in the mind *automatically* and *fast* (and will finally become *unconscious* and *intuitive*), as they are carried out in the minds of native speakers.

This example illustrates how it is possible to develop correct and fast unconscious and intuitive mental processes via initially learning them as conscious algorithmic processes, with their subsequent internalization and automatization through specially organized exercises.

Why some children become good students and problem solvers while most others don't

We have shown above that a large variety of tasks and problems objectively *require* (whether we realize it or not) an algorithmic process for their performance. We have already noted that, as a rule, teachers don't teach non-mathematical algorithms because, being largely unaware of their own algorithmic processes, they themselves don't know these algorithms. And, as a rule, teachers are not taught algorithms in the course of teacher training.

Why then some children become good students and problems solvers, while most others don't?

It is because, as was mentioned earlier, some children are able to *discover by themselves* – through trial and error – the required algorithmic processes.

Moreover, many of them are able to discover not only specific algorithmic processes but *general methods of discovering* those algorithmic processes as well. As with all methods, the general methods of discovering algorithmic processes consist of mental operations. The characteristic features of these operations are these: (a) they are specific "discovery operations", and (b) these operations are of a high level of generality. (We can also say that these are operations of higher order, or *meta-operations.*)

This explains why having discovered the general "discovery operations", high achievers can now discover algorithmic processes in *any discipline* and apply them to successful learning and problem solving *in any area of activity.* Those general discovery operations make up, according to our theory, *general intelligence.*

However, the majority of children (and adults) are unable to discover algorithmic processes and the methods of discovering algorithms on their own. As a result, they are unable to successfully learn and solve problems. They often remain poor students and poor problem solvers for the rest of their lives.

How to make all children good students and problem solvers

The solution follows from what has been said in the previous section.

To be able to make all children good students and problems solvers, we need to:

1. Uncover unconscious algorithmic processes of expert learners and expert problem solvers, and describe those processes explicitly.

2. Uncover general methods of discovering algorithmic processes and algorithms and describe them explicitly.

3. Teach students the algorithms of effective learning and problem solving, as well as the general methods of discovering algorithms on their own.

To be able to do all this, we have to know the *how to* of the above.

Specifically:

1. *How to* uncover unconscious algorithmic processes of expert learners and problem solvers, and describe them explicitly.

2. *How to* uncover the general methods of discovering algorithmic processes and algorithms, and to describe those general methods explicitly.

3. *How to* teach students the algorithms of effective learning and problem solving, as well as the general methods of independent discovery of algorithms.

What is Landamatics

Initially called the algorithmico-heuristic* theory and method of performance, learning and instruction, it was later dubbed *Landamatics* by the American scholars. This short designation was then repeated in many publications, including the "Encyclopedia of Psychology".**

As a theory:

1. Landamatics analyzes and explains the mental processes which underlie expert performance, learning and instruction.

2. It also defines specific ways of purposeful and accelerated development of such processes in students and adults through a special algo-heuristically-based course of instruction.

As a method, it represents a system of techniques of how to:

1. *Get inside* the minds of expert performers, learners and thinkers, and *uncover* those unobservable mental processes that underlie experts' ability to perform and learn as experts – the processes of which they themselves are largely unaware.

2. *Break down* those processes into relatively *elementary* (simple) component operations.

3. Explicitly *describe* those operations and their systems as algorithms or heuristics which will indicate to students and non-expert performers what they should do in their minds, and physically when needed, in order to be able to learn, perform and solve problems as well as the experts do.

(Those algorithms and heuristics serve as a tool of *transferring* the mental processes of experts to the minds of students and inexpert performers, these tools allowing teachers and trainers to *replicate* the mental processes of experts in the minds of students and non-experts.)

4. *Design* effective methods of teaching experts' mental operations and processes to students and non-experts.

5. *Create* a new type of instructional and learning materials – *algo-heuristically-based* materials, including textbooks and software, which will teach – purposefully, systematically and reliably – not only expert knowledge but experts' mental processes (algorithms and heuristics) as well.

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*A discussion of the heuristic aspect will follow.

**"Encyclopedia of Psychology", vol. 2, p. 277-278, John Wiley & Sons, 1984.

In short, Landamatics can be defined as a theory and method for the creation of expert performers, expert learners and teachers in a systematic, reliable and relatively fast way, without the need for students and novices and inexpert workers to go through years of conventional experience to become experts.

On the heuristics aspect of Landamatics

Not all tasks and problems are algorithmic in nature. There are problems for which algorithms are not known and cannot be created. Such problems are *heuristic*, or *creative*. (*Heuristic* and *creative* are synonyms; we will be using these terms interchangeably.) For their solution, heuristic problems require mental *operations of search*, and it is often not known where (in which field of tangible objects or field of knowledge) to look for the objects which can provide a solution.*

As a method of getting inside the minds of experts, Landamatics can uncover not only the mental operations of *algorithmic thinking* but also the mental operations of *heuristic (creative) thinking*. Once the latter operations – and their systems – have been uncovered, prescriptions (sets of instructions) can be formulated as to what one should do in his/her mind, in order to more easily find a solution to a creative problem.

Those instructions can be called *heuristic instructions*, or *heuristics*.

In the way the algorithmic instructions can guide – *and be a tool of formation of* – the algorithmic mental processes, the heuristic instructions (heuristics) can guide – *and be a tool of formation of* – the creative mental processes.

The principal difference between algorithmic and non-algorithmic instructions lies in the degree to which they determine, or specify, the corresponding mental operation. Algorithmic instructions determine (specify) their corresponding mental operations *completely* and *unequivocally*. Non-algorithmic instructions contain a greater or lesser *degree of uncertainty* as to what one has to do in his mind in order to arrive at the solution to a problem.

The following comparison will show the difference between algorithmic and non-algorithmic instructions given to a detective trainee as to how to examine a scene of crime:

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* We have discovered that between algorithmic and heuristic processes there is an array of intermediate processes that we have called *semi-algorithmic* and *semi-heuristic*. However, the discussion of these processes goes beyond the objectives of this article. (These processes are described in sufficient detail in L. Landa's "Instructional Regulation and Control: Cybernetics, Algorithmization and Heuristics in Education", pp. 146-150).

NON-ALGORITHMIC	ALGORITHMIC
In order to find a criminal who committed a burglary: 1. Thoroughly examine the scene of the crime; 2. Isolate all the things that may serve as clues; 3.	In order to find a criminal who committed a burglary: 1. Thoroughly examine the scene of the crime and check whether the burglar: (a) Left footprints. If yes, do..... If no, (b) Left fingerprints. If yes, do..... If no, (c) Left cigarette butt(s). If yes, do..... If no, (d) Left cigarette ash. If yes, do..... If no, (e) Etc.

It is easy to see that the non-algorithmic instructions of the type: "Thoroughly examine the scene of the crime" and "isolate all the things that may serve as clues" contain a lot of uncertainties: "What specifically do I have to look at in the process of examination?", "What specifically can serve as clues?".

Algorithmic instructions do not contain those uncertainties: they indicate precisely what a detective should look at, isolate and consider as clues.

It is obvious that in order for a student to be able to follow non-algorithmic instructions, it is necessary to teach him – *prior to using non-algorithmic instructions* – those mental operations that are involved in executing non-algorithmic instructions. If those operations have not been formed, there will be no benefits from giving a student non-algorithmic instructions. He/she will not be able to follow them and to perform them.

A situation that can arise, had the prerequisite mental operations not been previously formed, can be described in the form of the following dialog between a teacher and a student:

Teacher: Examine the scene of crime thoroughly and isolate all the things that may serve as clues.

Student: How to examine? What should I look for in the scene of crime? What can serve as a clue?

Why schools fail to effectively teach not only the algorithmic but the heuristic (creative) thinking as well

The above example has shown that:

1. Non-algorithmic thinking is also made up of specific mental operations.

2. Those mental operations can be identified and taught.

3. In order to be able to perform non-algorithmic (in particular, creative) processes and follow non-algorithmic instructions, it is necessary to have developed, in the students' minds, sets of *prerequisite mental operations* which can be effectively taught through algorithms of different levels.

In other words, non-algorithmic (including creative) processes are *based* on previously developed specific mental operations that form a *foundation* for creativity.

As no physical building can be reduced to its foundation, creativity is not reduced to algorithmic mental operations that can be effectively formed through algorithms. On the other hand, as no building can be built *without* a foundation, creative processes cannot be effectively formed – and function – without the underlying algorithmic operations and processes.

At the beginning of this paper, it was shown that teachers – as any other experts – are largely unaware of their mental operations and processes and, as a rule, don't teach them to students in a purposeful and systematic way. This teachers' unawareness and failure to teach mental operations properly include both algorithmic and non-algorithmic processes.

Today, teaching of creativity consists primarily of one of the following:

A. Teachers give students creative problems without teaching them the specific mental operations of search which are needed to solve those problems. The hope here is that the students would *somehow* figure out the ways of finding a solution.

This method can be likened to teaching people how to swim by throwing them in the water. Some people would discover – by trial and error – the operations of swimming and thus learn how to swim, others – the majority – wouldn't discover such operations and would never learn how to swim. It is important to have in mind that to discover the

physical operations of swimming is incomparably easier than to discover mental operations of creative thinking.

B. In addition to giving students creative problems, teachers give them, in the course of their attempts to find a solution, some instructions. However, these instructions are often so general and vague (like "look at the problem carefully", "try again" and the like) that they don't actually teach any specific operations of creative search and thus don't teach creativity.

As with the previous method, the capable students who are lucky to discover by themselves - through trial and error – the operations of creative thinking become creative. Those who fail to discover the operations of creative thinking on their own don't become creative despite all the efforts of teachers to develop their creativity.

C. When students encounter a difficulty in finding an idea which would lead to a solution, teachers, willing to help students, simply impart this idea to them (like "try to use this theorem...", "look at this element in the diagram" etc.). This method provides students with solutions instead of teaching them how *to arrive* at solutions and thus simply eliminates the need in creative thinking. With this method, the students would learn *specific* solutions to *specific* problems, but not the *general operations* (methods) *of searching* for solutions to be applied to *any* problem.

In neither of the indicated methods, teachers, as a rule, are aware of the algorithmic mental operations which are to be developed in the minds of students prior to teaching them heuristic operations and which form the foundation for the students' ability to understand, follow and make use of the "good" heuristics which the teacher may use.

An analogy. Giving detectives, in the course of their training, a heuristic instruction, "thoroughly examine the scene of the crime and isolate all the things that may serve as a clue", would be useful only if, prior to this, they were taught the things (objects) which can serve as clues *as well as* the operation of examining them. Without this preliminary algorithmic instruction which would develop in trainees the prerequisite knowledge and mental operations, the given heuristic would be almost of no use.

By uncovering mental operations underlying creativity and equipping teachers with methods of explicit teaching those operations, Landamatics enables teachers to form creative processes and abilities in all students in a purposeful, systematic and reliable way.

An example of explicit teaching mental operations of creativity using the Landamatics method

The following experiment to uncover and teach some mental operations underlying creative thinking was conducted by us:

Students were seated – individually, each at a time – in a room containing a variety of different objects: several books, a telephone, pencils, a ruler, a trumpet, a guitar, a radio, a painting, a bottle with water, and a number of other items.

They were given a piece of cheese and asked how they would cut it evenly.

Here is a fragment of a typical dialogue:

Experimenter: What would you do to cut this cheese evenly?

Test subject: I would use a knife.

Experimenter: There is no knife.

Test subject: Then I would use a wire.

Experimenter: There is no wire.

Silence. Thinking.

Experimenter: Would some objects available in the room be of any help?

Most of the students (8 out of 10) couldn't find a solution and gave up.

However, one of them went over to the guitar, took off the string, and cut the cheese. Another proposed to use the string and also said the ruler could be used. (it had a metal edge).

Let us reconstruct the mental processes that were going on in the minds of those students who didn't have creative thinking and therefore couldn't solve the problem:

The task of cutting cheese was associated in their minds, due to their past experience, with the knife as an object (tool) for performing the tasks. Whenever this task arose, it led to actuation of that association. But in the experiment a knife was not available. Then they began to activate another tool that was associated with the tasks of cutting cheese evenly: it was a wire. But the wire was also unavailable.

In the minds of eight students the store of associations between the task and the *objects* that could serve as tools for performing the task was exhausted. The mechanism of involuntary actuation of associations – when images of the tools that can be used came to the mind *by themselves* – didn't work any longer. They had to start actively *doing* something in their mind to find a solution but they did not know what: they did not know what *mental operations* to perform. and gave up.

Those two students who found the solution initially went through the same mental process as the majority of the test subjects – they wanted to use the objects (a knife and a wire) that were suggested by their associations. When, however, it turned out that those objects were unavailable (i.e., it became clear that the associations didn't lead to the solution), they *began executing certain mental actions* which finally led to finding objects-solutions. These objects (a guitar string and the ruler) were *not components* in the network of associations which was formed in their past experience and which gave the solution to the problem.

What did they *do* in their minds in order to *break out* from the network of formed associations and thus *overcome* the binding - and limiting – power of the past experience?

Here is our reconstruction of the mental operations which they performed in their minds:

1. They asked themselves (explicitly or implicitly) what *attribute* of the objects which are normally used as tools (a knife and sometimes a wire) for cutting soft objects like cheese was responsible for doing the job.

2. They answered to themselves - sharpness.

3. They then *isolated* this *attribute* in their mind (isolation is a mental action).

4. They then began to *scan* the objects in the room (scanning is a mental action of *isolating* one object after another) and *checking* each isolated object *for the attribute of interest*, i.e. sharpness.

(Checking objects for some attribute are mental actions of *superimposing* the image of the attribute on an object and *seeing* whether the object has the attribute).

5. If the isolated object had the attribute of interest, this object was selected as a solution to the problem.

6. If the isolated object didn't have the attribute of interest, they *isolated the parts of the object* (another mental action) and *checked each isolated part* (like a string on the guitar) *for the attribute* of interest (another mental action).

The execution of those mental actions in a systematic way led the two students to stumbling upon the guitar's strings that had the sought-for attribute of sharpness. (It was the first solution to the problem.) The second of the two students continued the execution of the above mental actions after he had found the first solution and stumbled upon the second object that has the required attribute of sharpness - the ruler with the metal edge. This was the second solution to the problem.

The just described sequence of mental actions was our hypothesis about the makeup (anatomy) of the process of creative search for the given type of problems.*

Now it was necessary to verify the hypothesis.

The method of verification we used can be described as follows:

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*The problem of finding a tool to do something when the customary tool(s) are not available belongs to an extremely wide class of heuristic (creative) problems encountered both in the everyday life and in science and technology.

(a) formulate heuristic instructions as to what one should do, according to the hypothesis, in his/her mind – i.e. *which mental operations to perform* – in order to find a solution to this kind of problem;

(b) select students, who failed to solve the problem, and teach them the above heuristic operations via formulated heuristics (the teaching must be done on objects having no resemblance with the objects used in the test problem);

(c) thereafter, offer the students the original problem that they failed to solve earlier and see if they now can solve it.

> If they could, it is most probable that the hypothesis about the mental operations involved in this creative process was correct.
>
> If they could not, then either the operations discovered were incorrect or incomplete, or the method of teaching those operations was ineffective.

Here are examples of heuristic instructions (heuristics) which were formulated on the basis of the discovered operations:

If customary objects (tools) of performing a tasks are unavailable, then:

1. Ask yourself what *attribute* of the objects normally used for performing the task are responsible for the ability to achieve the goal.

2. *Isolate* this attribute in your mind.

3. Start *isolating (scanning) objects* that are available *one by one.*

4. In each isolated object, *check to see* if it has the attribute you are looking for.

   If yes, this object is the solution to the problem.

   If no,

5. In each isolated object, isolate its parts (one by one) and check to see if the isolated part has the attribute you are looking for.

Etc.

It is easy to see that the set of these heuristic instructions (heuristics) represents a general prescription - *a general method* – as to what one has to *systematically do* in his/her mind in order to find a creative solution to *any* of the creative problems of this type. The psychological function of each instruction (heuristic) consists in *actuating* a proper mental operation and its *execution.*

This is how operations making up the creative processes are formed via heuristics:

Students initially use and follow *external* heuristic instructions (for example, written on paper) which actuate the required mental operations *from the outside.*

In the process of following and applying those external heuristics, they learn and *internalize* them.

As a result, they become able to give those instructions to themselves – the external instructions become internal *self-instructions*. The need for external instructions vanishes.

Finally, after the mental operations actuated initially by the external and then by the internal instructions (heuristics) get well practiced, learned, and automatized, the need even for self-instructions vanishes. The proper mental operations become directly activated by the problem situations themselves rather than by self-instructions. The heuristics which played a critical role in actuating and learning the appropriate mental operations become unneeded and finally, as a rule, get forgotten.

After the heuristic prescription as to what one should do in his/her mind to find a solution to this kind of creative problems was designed, an experimental instruction in the operations of creative thinking was conducted with those students who failed to solve the problem in the original test.

Experimental instruction

Eight students who initially failed to solve the problem were taught – in a operation-by-operation manner – what they should do in their minds in order to find a solution to a problem of this type.

1. The heuristics – one by one - were explained to them.

2. Each heuristic was illustrated by examples not related to the problem of finding a way of cutting cheese.

3. Some exercises for practicing each of the mental operations involved were conducted (for example, an exercise in isolating attributes of an object; an exercise in isolating parts of an object and checking each part for some given attribute, etc.). The objects used in those exercises were not related to the task of finding a tool to cut a piece of cheese.

In the course of explaining each heuristic, students wrote down those heuristics. By the end of the experimental instruction, they had in writing the whole heuristic prescription. The heading for the prescription was "What you should do in your mind in order to find an unconventional object to solve a problem".

The experimental instruction lasted about 90 min.

Control test and test results

After the instruction was completed, the students were given the same problem of how to cut cheese in the absence of knife or wire – the problem they failed to solve before experimental instruction.

Seven students out of eight (88%) were now able to find a solution to this creative problem. Four students out of seven (57%) were able to find two solutions.

We must note that we trained students in the general method of searching for an unconventional *tangible* object that had a sought-for attribute. But creative people are able to do the search among *images* of objects as well. With some modification of the method of instruction used, students can be taught (and trained) to do the search among the *images* of objects, although this kind of search is more difficult than the first one.

Effectiveness of Landamatics in teaching and learning of different disciplines

Here are some examples of the effectiveness of Landamatics as compared to the effectiveness of the conventional methods of instruction. The results cited below were achieved in instruction conducted either by L. Landa or his students, disciples or associates. Instructional objectives included teaching students either algorithmic or non-algorithmic methods of thinking depending on the nature of the subject matter and the tasks involved.

Subject matter	Abilities tested	Conventional method	Landamatics method	Times better
Mathematics (geometry)	Ability of junior high school students to solve problems of proof	25% of test problems were solved after 1 1/2 years of conventional instruction	87% of test problems were solved after 7 hours of algo-heuristic instruction	3.5 times
Physics	Ability of junior high school students to solve more than 40% of problems of average and high level of difficulty	None of students	88% of students	88/0 times
Grammar of native tongue	Ability of junior high school students to identify the type of a sentence	Error rate 29.5%	Error rate 6%	4.9 times
Foreign language (English)	Ability of college students to comprehended 80 to 99% of sentences in a complex written scientific text	3.2% of students	88.5% of students	27.6 times
Medicine	Ability of 6th year medical students to solve diagnostic problems in radiology	Error rate 64%	Error rate 4%	16 times
Theory of music	Ability of college students to perform assignments requiring application of complex music-related concepts	Failure rate 34%	Failure rate 7%	4.8 times

A brief summary of selected principles of algo-heuristic method to instruction

1. Before starting to teach any topic in any subject, it is necessary to identify not only knowledge and practical actions involved in learning and problem solving but mental operations as well.

2. Before starting to teach how to solve any kind of problem, it is necessary to identify whether problems of this type are algorithmic or non-algorithmic in nature.

3. If problems are algorithmic, it is necessary to find, design, and formulate algorithms of their solution. If problems are non-algorithmic, appropriate heuristics are to be found and formulated.

4. It is much more important to teach algorithmic and heuristic *processes* rather than algorithmic and heuristic *prescriptions*. Algorithmic and non-algorithmic instructions (prescriptions) are just effective *tools* for teaching and learning knowledge and mental processes. Once the appropriate process has been formed with the help of tools, the tools are no longer needed.

5. Forming thought – and other mental – *processes* must be based on purposeful and systematic teaching of the mental *operations* which make up those processes.

6. Before starting to teach any mental operation, it is necessary to make sure (via special diagnostic tests) that the prerequisite mental operations have already been formed, internalized and automatized. If they haven't been, start with forming the prerequisite operations.

7. Learned mental operations shall be combined in systems (blocks). Those blocks are to be gradually enlarged by adding to them new operations and sub-blocks of operations. Such enlargement of blocks of operations will lead to forming more and more complex *systems* of operations (i.e. mental *processes*) which make up *methods of thinking*.

8. If students have difficulties in learning and performing certain mental operations *simultaneously*, it is important to teach those operations *sequentially*, on an operation-by-operation basis, gradually *combining operations into blocks* and thus transforming sequential operations into simultaneous.

9. It is much more important – and educationally valuable – to teach students how to *discover* algorithms and heuristics *on their own* rather than simply *provide* them with *ready-made algorithms and heuristics*. However, because discovery of algorithms and heuristics is a time-consuming process, it is impractical to get students to discover *all* the algorithms and heuristics they have to know and master. The discovery approach should be combined with teaching students "ready-made" algorithms and heuristics already discovered in science or by instructional scientists or teachers.

10. Students don't have to *memorize* algorithms and heuristics. Algorithmic and heuristic instructions should be used initially by teachers – and then by students themselves – as *activators* of appropriate mental operations and their systems (i.e. processes). The mastery and internalization of operations will occur in the course of their application to solving carefully designed problems that require those operations.

In this paper, only some aspects of the algo-heuristic theory and method of instruction were covered. The scope of the theory and method is much greater. They include, for example, such issues as: (a) how the algo-heuristic method can be used to increase the effectiveness of teaching and learning *knowledge*, not just *mental processes*; (b) how to adapt the method to individual characteristics of students; (c) how to create algorithms for *expert teaching*, not just for expert *learning* and *problem-solving*. Expert level algorithms for teaching (i.e. instructional algorithms) make it possible to *replicate* in novice and less experienced teachers the mental and physical operations of expert teachers, and thus turn the novice teachers into experts in a much shorter time – without the need for years of conventional experience. Today novice teachers need those years of experience in order to discover – mostly through trial and error – the unconscious algorithms of expert teachers and start using them. And not all teachers are able to discover those algorithms on their own.

With the algo-heuristic approach to teacher training, the superior algorithms of expert teaching can be *transferred* to teacher students and novice teachers in a *systematic* and *reliable* way and thus ensure that the majority of teachers would be able to perform at a near expert level by the end of training rather than as a result of years of experience.

A more detailed discussion of the issues covered in this article can be found in the following publications:

L. N. Landa, *Algorithmization in Learning and Instruction.* Educational Technology Publications, Englewood Cliffs, New Jersey, 1974.

L. N.. Landa, *Instructional Regulation and Control: Cybernetics, Algorithmization and Heuristics in Education.* Educational Technology Publications, Englewood Cliffs, New Jersey, 1976.

L. N. Landa, The Algo-Heuristic Theory of Instruction. In: Ch. M. Reigeluth (Ed.), *Instructional-Design Theories and Models: An overview of Their Current Status.* Lawrence Erlbaum Associates, Publishers, Hillsdale, New Jersey, London, 1983.

L. N. Landa, A fragment of a Lesson Based on the Algo-Heuristic Theory of Instruction. In: Ch. M. Reigeluth (Ed.), *Instructional Theories in Action: Lesson Illustrating Selected Theories and Models.* Lawrence Erlbaum Associates, Publishers, Hillsdale, New Jersey, London, 1987.

L. N. Landa, The Improvement of Instruction, Learning, and Performance: Potential of Landamatic Theory for Teachers, Instructional Designers, and Materials Producers. An Interview with L. Landa. *Educational Technology*, October 1982 and November 1982.

L. N. Landa, Landamatics Ten Years Later. An interview with Dr. L. N. Landa. *Educational Technology*, June 1993.